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Peirce's NonReduction and Relational Completeness Claims in the Context of First-Order Predicate Logic

Interdisciplinary Seminar on Peirce*

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Herzberger (1981) gave a systematic analysis of Peirce's relational theses, and included work on "bonding algebra." We approach the topic from a different expository angle, and discuss its basics with a less weighty toolbox. Setting forth the fundamentals in Sections **I** and **II**, we then mention in Section **III** the availability of an embedding of our basic set-up in a type of "fine structure" for relational statements of higher adicity. This embedding consists in using monadic, dyadic, and triadic "atoms" (which we call "Relation Types") to construct "molecules" of arbitrary adicity via bonding. These molecules can then be incorporated into an easy-to-read version of Beta graphs that can readily be made to embrace first-order predicate logic. Possibilities for cross-disciplinary applications are noted.

Introduction

On many occasions, Charles Sanders Peirce proposed the following theorem of logic[1]:

NonReduction Theorem (**NRT**): *From resources consisting only of monadic and/or dyadic relation types and the operation of* ***Bonding***,[2] *it is not possible to construct triadic relations.*

For a number of years, beginning during Peirce's lifetime, this theorem has been subject to various levels of informed doubt.[3] For example, Herzberger (1981, 41) regretted that

> Notwithstanding Peirce's long-sustained efforts to establish his thesis, no cogent demonstration has so far been found in any of his writings, and many scholars have long since written it off as one more uncollectable debt.

References to the **NRT**, other than by Peirce himself, were for a long time scarce, perhaps in part because of this factor.

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However, in more recent times, that skeptical attitude has been challenged, and Peirce's thesis has been explicated and supported by a growing group of scholars, beginning with Herzberger (1981), then Ketner (1986), Ketner (1989: the bibliography cites other relevant works), Burch (1991), Peirce (1992 [1898]), C. Hendrick (1993), Christiansen (1993), Fernandez (1993), in Houser et al. (1997) several essays (Burch, Brunning, Anellis, plus the Foreword by Hintikka and Hilpinen), Dau (2003), Dau and Correia (2006), Correia and Poeschel (2006), Correia (2008) as well as McCurdy 2011.[4]

One theme we have detected-concerning searches in the Peirce corpus for his defense of the **NRT**-is that scholars have been looking in terms of formats, notations, or logical systems currently in use. Our efforts, and those of many others beginning with Herzberger, are directed toward a study of Peirce's graphical or topological logic as the probable locus of his justification of his reduction thesis. Thus, we find a growing number of scholars referencing **NRT**. Through our study of Peirce's graphical work we think we have arrived at a better understanding of his thesis. Furthermore we believe we can provide a readily accessible version of our result with a minimal apparatus and present it in such a manner as to be available to virtually any scientific discipline. We think this overcomes contemporary complaints that Peirce's studies are eccentric, inaccessible, or nonstandard. Our attempt to offer a basic form for the result can aid in drawing further consequences from it; and thereby, we hope to encourage the deployment of Peirce's result into relevant areas of study. We also regard this essay as an initial contextual frame for our continuing research in Peirce's graphical/topological logic, especially in regard to its fundamental role in establishing Semeiotic. The details of this larger development, as noted in the postscript, are too numerous to include in this short article; but they appear to allow for a wide variety of extralogical applications, in areas ranging from modeling neuronal circuits in the brain, to studying networks of interpersonal communication, to visual perception and mental imagery studies, to parts of physics (see, for example, Beil and Ketner [2006]); and they do not involve quite as much heavy technical machinery as do, say, Burch's PAL and the more traditional ways of writing Beta graphs. For the present we content ourselves with the fundamentals. But to be just a bit more specific at the outset, we close this introductory section with the following remarks.

Herzberger's article (1981) provided a systematic analysis of **NRT** and some related matters, and included work on "bonding algebra." Here we approach the topic from a somewhat different expository angle, and discuss its basics with a bit less weighty toolbox than he used. Having set forth the fundamentals in sections **I** and **II** in what we believe to be a highly digestible form, we then mention, in section **III**, the availability of an embedding of our basic set-up in a kind of "fine structure" for relational statements of higher adicity. This embedding consists in using monadic, dyadic, and triadic "atoms" (what we are here calling "Relation Types") to construct "molecules" of arbitrary adicity via bonding. These molecules can then be incorporated into an easy-to-read version of Beta graphs that can readily be made to embrace first-order predicate logic. (The molecules themselves are capable of giving, under a suitable semantics, extensional representation to relations of arbitrary adicity in arbitrary domains.)

I. The NonReduction Theorem [NRT]

The First-Order Predicate Logic with Relations and Identity, as given in Irving M. Copi *Symbolic Logic* (1979), is herein referred to as C. Following Peirce, a ***Relation*** is understood as a fact about some number of items (see also, Copi 1979, 116–117). Thus a monadic relation is a fact about one item, a dyadic relation is a fact about two items, and a triadic relation is a fact about three items. Of particular interest to us are factual relations arising from scientific inquiry.

Generalized monadic, dyadic, and triadic relation types will be expressed in the manner of C in these general forms:

Monadic relation type	M*u*
Dyadic relation type	D*vw*
Triadic relation type	T*xyz*

where *u, v, w, x, y, z* are individual ***Placeholders***, each of which may also be designated as a ***Relate*** of the relation to which it is attached, and M, D, T, are each relation predicate types of the kind indicated. These may be read in English in the following manner.

M*u*: an unspecified *u* is in an unspecified monadic relation M (and in like manner);
D*vw*: *v* is in a dyadic relation D with *w*;
T*xyz*: *x*, *y*, and *z* are in a triadic relation T.

Each of these three – M*u*, D*vw*, T*xyz* – are generalized propositional function types which are not now quantified. We see that a token of each of these three types would be some specific relation with the appropriate number of placeholders for that type, as follows.

Type	**Token**	**English (Peirce's Rhematic Form)**
M*u*	Red*u*	__ is Red.
D*vw*	Caused*vw*	__ Caused __
T*xyz*	Promised*xyz*	__ Promised __ to __

A placeholder can be either open or closed (but not both). An ***Open*** placeholder is available to receive a specification. A placeholder is ***Closed*** if it has been bonded with another placeholder, or has been supplied with a constant (or, in other words, a definite individual, such as "the one and only fifth King of France"). Peirce illustrated this aspect through what he called ***Rhemata***,[5] shown in the third column above. In rhematic analysis, a blank '____' is used as notation representing a placeholder, while the difference of location of multiple placeholders (as in dyads or triads) within each ***Rheme*** is used to indicate different placeholders (a function which in C is served by variables *u, v, w, x, y, z*).

Each of these relation types possesses a property Peirce identified as ***Valency***, thus: the valency of M is one because it has one open placeholder, marked by *u*; the valency of D is two because it has two open placeholders, marked by *v, w*; the valency of T is three because it has three open placeholders, marked by *x, y, z*. The ***Adicity*** of a relation type, as opposed to its valency, is the number of mutually distinct markers (*u, v, w, x, y, z*) that appear in open placeholders. For example D*vv* has valency two, but adicity one.

A ***Graph Presenter*** may select some list of relation types from resources typified by M*u*, D*vw*, T*xyz*. When that list for consideration or examination is mutually agreed to with a ***Graph Evaluator***, that list is denoted the ***Graph Under Consideration***, or simply, ***The Graph***.[6] If a placeholder becomes closed, it is no longer counted in determining the valency of the graph at hand.

When we wish to present a graph for consideration which is composed of more than one relation type (including repetitions of the same relation type), we will write the graph list using these marks '][', which may be read as "Let us agree to consider the items on either side of][as the parts of the graph under examination"; thus, we might wish to begin a consideration of M*u* together with T*xyz*. We could then write M*u*][T*xyz* . If a third item is to be part of the graph being considered, we would write M*u*][T*xyz*][D*vw*, which means that the three relation types listed are designated as the graph under consideration. We note that][is not identical to any of the standard operators in C such as **&**, or **V**, or ⊃, or ⇔ (Conjunction, Inclusive Disjunction, Material Implication, Material Equivalence). The symbol][serves solely as a collector and has no formula formation role.

Now we define the operation of bonding. If, where $\mathbf{R}_1$][$\mathbf{R}_2$ are specific relations from any of the three types mentioned above, and where ϕ is some relate (placeholder) of $\mathbf{R}_1$ and θ is some relate (placeholder) of $\mathbf{R}_2$, then an operation **B**, ***Bonding***, on $\mathbf{R}_1$ and $\mathbf{R}_2$ may be defined (':=' is understood to mean "is defined as").

($\mathbf{R}_1$ **B** $\mathbf{R}_2$) := the following process:

Step ***a***. From the resource types M*u*, D*vw*, T*xyz* select one type (or a token of that type) to be designated as $\mathbf{R}_1$;

Step ***b***. From the resource types M*u*, D*vw*, T*xyz* select a second type (or a token of that second type) to be designated as $\mathbf{R}_2$ – we now have the Graph $\mathbf{R}_1$][$\mathbf{R}_2$ under consideration;

Step ***c***. Select one placeholder from $\mathbf{R}_1$ and designate that placeholder as ϕ;

Step ***d***. Select one placeholder from $\mathbf{R}_2$ and designate that placeholder as θ.

Step ***e***. Merge (fuse or "weld"[7]) ϕ and θ; this closes both ϕ and θ, thus decreasing the after-bonding valency count by two, and removes their markers.

Step ***f***. Using ψ to indicate the two placeholders that are now closed and have been joined and stripped of their markers, we rewrite $\mathbf{R}_1$ and $\mathbf{R}_2$ to show the changes made in step ***e***; this rewritten form is the result of the operation $\mathbf{R}_1$ **B** $\mathbf{R}_2$.

An example will illustrate the process. Suppose as $\mathbf{R}_1$ we select a specific dyad G*xy* and as $\mathbf{R}_2$ we select a specific triad S*uvw*. For ϕ we may select either *x* or *y* (suppose we select *y* for whatever reason), and for θ we may select either *u*, or *v*, or *w* (suppose we select *v*); to complete the operation we rewrite the relational expressions G*xy* and S*uvw* to show that the chosen ϕ and θ are now, after bonding, specified as the same placeholder ψ (where '=>' means "yields the constructive result"), thus: Given the graph G*xy*][S*uvw*, a possible bond is:

G*xy* **B** S*uvw* => G*x*ψ][S*u*ψ*w*.

The post-bonding complete graph is a different graph, but its status is still that of a "graph under consideration"; hence the resulting graph continues to be written with the collector marker ']['. No *assertion* (*statement, proposition*) has yet been presented.

Notice that in the above operation, initially, prior to performing the bonding operation, all placeholders involved, marked by *x, y, u, v, w* were open, that is, were available for designation. Each bonding operation will always close two placeholders such as ϕ and θ by specifying them to be joined and stripped of their markers. This result is shown in the expression G*x*ψ][S*u*ψ*w* above, wherein two placeholders, with the two previous markers *y* and *v*, are no longer open, but are now closed and designated as the same (welded) placeholder ψ as a result of the bonding activity. This example shows only one of several bonding operations that could be performed on the two relations mentioned, simply by varying the selection of ϕ and θ.

If we illustrate this operation with an example using Peirce's rhematic analysis notation, the operation of bonding can perhaps be further clarified. Let us select "___ *Struck* ___" as $\mathbf{R}_3$, and "___ *is Red*" as $\mathbf{R}_4$, that is:

$$\_\_\_\ \textit{Struck}\ \underset{\phi}{\_\_\_} \qquad][\qquad \underset{\theta}{\_\_\_}\ \textit{is Red.}$$

For ϕ we select the right hand blank in $\mathbf{R}_3$, and since $\mathbf{R}_4$ has only one placeholder, we take that as θ. This bonding operation would proceed as follows:

$$\mathbf{R}_3\, \mathbf{B}\, \mathbf{R}_4 \qquad => \qquad \_\_\_\ \textit{Struck}\ \underset{\psi\ \psi}{\_\_\_\_\_\_}\ \textit{is Red.}$$

This would read: "Whatever we might eventually specify as the left-most placeholder of $\mathbf{R}_3$ *Struck* the right-most placeholder (whatever it might be) of $\mathbf{R}_4$ which *is Red* ($\mathbf{R}_4$).

If we ask what bonding patterns might be possible given the three relation types M, D, T, we find that there are only eight possible combinations (where '**A**' means "this type is available for use in bonding" and '**U**' means "this type is unavailable for use in bonding") as summarized in the table below.

	M	D	T
Case 1	A	A	A
Case 2	A	A	U
Case 3	A	U	A
Case 4	A	U	U
Case 5	U	A	A
Case 6	U	A	U
Case 7	U	U	A
Case 8	U	U	U

Table 1

Notice in Case 4, only Monads are available, so any bonding operation attempted within this case would involve two or more Monads. Similar remarks apply to Case 6 which involves only Dyads, as well as to Case 7 which involves only Triads. Now we are ready to give a straightforward derivation of **NRT** in our context:

> *From resources consisting only of monadic and/or dyadic relation types* M, D *and the operation of* ***Bonding****, it is not possible to construct triadic relations.*

In each of the above listed eight cases, in which the operation **B** is applied to two available selected relation types, if there is no case in which triadic relation types can be produced or constructed using resources composed only of monadic and/or dyadic relation types, then **NRT** will be established.

(I-1) T disallowed as a resource: Any case that allows use of a triadic relation type as a part of the construction resources is rejected due to violation of the conditions of the **NRT** hypothesis, which specifies that the resources used in bonding operations be composed exclusively of monadic and/or dyadic relation types.

(**I**-2) M **B** M: A monadic relation type bonded with another monadic relation type produces only relations with zero valency (a condition in which there are no more open candidates for bonding within the relations available). Each of the two monads selected has one placeholder. The operation **B** will consume exactly those two, so the valency of the resulting expression will be zero.

(**I**-3) M **B** D: Monadic relation types bonded with a dyadic relation type produces only monadic relation types. A dyad has two placeholders available and a monad has one. Any possible way of applying B will produce only one remaining open placeholder.

(**I**-4) D **B** D: One dyadic relation type bonded with another dyadic relation type produces only dyadic relation types. Given two dyads, there are four placeholders available on which to use the operation **B**. No matter which combination of two placeholders are selected for **B**, afterwards only two placeholders will remain open, which means that the valency of the result will be two.

(**I**-5) Consideration of cases:
Case 1. Rejected, T disallowed.
Case 2. Rejected, M **B** D.
Case 3. Rejected, T disallowed.
Case 4. Rejected, M **B** M.
Case 5. Rejected, T disallowed.
Case 6. Rejected, D **B** D.
Case 7. Rejected, T disallowed.
Case 8. Rejected, because there are no resources, hence bonding cannot occur.

(**I**-6) THEREFORE: **NRT** holds for all bonding operations carried out within the resource pool consisting of M, D.

Before leaving the topic of **NRT**, we should mention three common relations, one falling under type D and two others under type T, which would therefore be governed by the eight possible cases in Table 1.

(1) ***Bi-Identity:*** Common identity, such as $x = y$, we shall designate as bi-identity and abbreviate as $\mathbf{I}_2$. It is clearly a token of type D. In rhematic notation it would appear as '___ *equals* ___' whereas in the notation we are using it would appear as '$\mathbf{I}_2xy$'.

(2) ***Teridentity:*** is a token of type T, and would be written rhematically as ' ___, ___, ___ *are at once equal* ', or in our notation as '$\mathbf{I}_3uvw$'.

(3) ***Transitivity:*** It is true that $\mathbf{I}_3$ can be constructed from $\mathbf{I}_2$ using the relation of transitivity (which shall be displayed in our notation as $\mathcal{T}$). But what is transitivity? Copi (1979, 135) explained it in this manner (where ' $\supset$ ' is the symbol of material implication in C):

> A *transitive* relation is a relation such that if one thing has it to a second, and the second to a third, then the first must have it to a third. A propositional function 'Rxy' designates a transitive relation if and only if $(x)(y)(z)$ [(Rxy & Ryz) $\supset$ Rxz].

Thus we see that another common relation, transitivity, is itself a token of relation type T, a fact about three items, and thus falls under the eight possible cases in Table 1. That is, the construction $\mathbf{I}_2][\mathcal{T} \Rightarrow \mathbf{I}_3$ would be an instance of Case 5 of Table 1, which is disallowed.

II. The Relational Completeness Theorem [RCT]

Often in the literature, this theorem is mentioned as Peirce's Completeness Theorem; however, to avoid confusion with various other well-known completeness theorems, we choose to employ the phrase shown above, which is a way to state that all relation types in terms of valency can be constructed from resources containing only monadic, dyadic, and triadic relation types and the operation of bonding.

(**II**-1) Valencies of Four or Higher: From resources consisting only of triadic relation types, the operation of bonding can produce relation types of valency four, and of relation types of any valency greater than four.

In the foregoing discussion of **NRT** we have considered all bonding patterns except T **B** T. This was excluded due to the necessity of attempting to use only monads and/or dyads with bonding to form a resultant triad, an attempt which failed.

Having established **NRT**, if we now examine the above pattern of bonding two triadic relation types, we find that valencies equal to or greater than four may readily be produced.

(**II**-2) Through simple construction, we find that:
(a) T **B** T will produce a result of valency four; moreover,
(b) (T **B** T) **B** T will produce a result of valency five; and
(c) [(T **B** T) **B** T] **B** T will produce a result of valency six.

In (b) the parentheses indicate that the left bonding is to occur first, followed by a second bonding of that result with the third triadic relation type; the use of square brackets in (c) indicates a continuation of that ordering convention.

(**II**-3) This sequence, (a), (b), (c), serves as the basis for a mathematical induction.

(**II**-4) So, in general, a graph of valency n, where n is equal to or greater than four, can be produced, given $n - 2$ triad types and $n - 3$ bonding operations.

(**II**-5) Valencies of one, two, and three are directly available in the given resources consisting of monadic, dyadic, and triadic relation types. Further, we have shown that from only triadic types, we can produce all higher valencies.

> THUS, **RCT** holds in the context of M, D, T and bonding: *Given the resource pool of monadic, dyadic, and triadic relation types* M, D, T, *and the operation of bonding* B, *one can produce relational forms of any valency.*

To summarize:

(**I**) Triadic relational types cannot be constructed using bonding and a resource base consisting only of monadic and/or dyadic relation types (**NRT**); and

(**II**) Relation types of any valency can be constructed using bonding and a resource base consisting of monadic, dyadic, and triadic relational types (**RCT**).

III. Postscript

The present paper is somewhat of a framework, not showing much of what can be built on top of the basic architecture it presents. Our goal has been simply to define and discuss an elementary but effective triadic approach, from which one can build a version of Beta that we think is both easier to manipulate and more precisely tuned at its ground-floor level than are the classical versions most often found in the literature.

Although space prohibits any detailed discussion here, we can use bonding in our sense to start from monads, dyads, and triads as "atoms," use them via bonding to produce "molecules," and subsequently use these molecules to provide *purely extensional* representations of higher-adicity relations; this molecular setup can be embedded in a version of Peirce's Beta Existential Graphs that is easier to read than the usual formulations in the literature. Once endowed with suitable axiom and inference schemes, Beta graphs are known to be one way of formalizing first-order predicate logic, which we have here mentioned in the form of **C**, since that is one of the standard approaches to it that people tend to be familiar with, whether they have ever seriously considered Beta graphs. And we can supply our version of Beta graphs with an observationally motivated semantics relative to which the following proposition (clearly a version of **NRT** for the "Teridentity" relation, relative to unquantified graphs) can be proven without much trouble: **Proposition**. There is no way, in our bond-incorporating version of Beta graphs, to (semantically) characterize the teridentity relation by means of a well-formed graph **G** such that

(a) every atom (and hence every molecule) occurring in **G** has valency less than or equal to 2, and

(b) the operations of bonding, conjunction, and negation are allowed in forming **G**, and

(c) valency (**G**) = adicity (**G**) = 3.

Any such characterization would have to exhibit either valency (**G**) greater than or equal to 4, or adicity (**G**) less than or equal to 2. (As an example, suppose we are given the expression '$\mathbf{R}_5xy$ & $\mathbf{R}_6z$', of valency and adicity 3, and we claim it to be a correct expression of Teridentity. Let D be a domain of discourse having at least two elements; and let d_1, d_2, be distinct elements of D. Now in D $\mathbf{R}_6z$ can only represent some subset of D; and $\mathbf{R}_5xy$ must clearly express Bi-identity, $x = y$. Obviously, $\mathbf{R}_6z$ cannot represent the empty set. Suppose, then, that a subset **D** of D is the set that $\mathbf{R}_6z$ represents in D, and let, say, $d_1 \in \mathbf{D}$. Then the following is true in D: $\mathbf{R}_5d_2d_2$ & $\mathbf{R}_6d_1$. But therefore it is not in fact the case that $\mathbf{R}_5xy$ & $\mathbf{R}_6z$ correctly expresses Teridentity; it fails to do so in any domain with more than one element. A full and rigorous proof of the Proposition proceeds by induction on the constructional complexity of an alleged teridentity-representing graph. Notice, by the way, that the expression '$x = y$ & $y = z$', when written graphically, has valency four.)

Of course, as documented by Burch (1991) there are many relations of adicity three that *can* be so characterized; they include Cartesian Products. Burch calls such relations "degenerate." The argument for the proposition breaks down, extensionally, if quantification is allowed. On the other hand, full relational completeness is achieved, extensionally, at the molecular level, using, of course, ternary atoms. It should also be mentioned that our approach does not provide the intensional analysis of Burch's PAL.

In conclusion, we wish to comment a little more fully on possibilities, briefly alluded to in the introduction, for extralogical use of the "triadic framework."

A good deal of effort has already gone into the application of "neural networks" to the study of brain circuitry; those networks are of a standard graph-theoretic character, whereas the "Peircean" approach of this paper, when fully developed, is not standard in that sense, but might well serve as an additional tool for such studies (see Rumelhart 1987 and Rumelhart and McClelland 1986).

In another direction, it was a somewhat underappreciated contemporary of Peirce, Christine Ladd-Franklin, who even then applied the concept of a triad-based relational system to questions of biological eye functions, and the perceptual functions of visual reasoning (Ladd-Franklin 1892, 1902). Perhaps there are questions concerning image-processing that could be framed and studied, to some advantage, by way of this approach.

On a somewhat larger scale, in our future research we wish to entertain **NRT** and **RCT** as governing the bonding of relations, no matter in what discipline such relations might be under study. To this end, we briefly describe the following working hypotheses as conjectures from our previous discussion.

Working Hypothesis 1: *Relations, under study within any discipline, are governed by **NRT** and **RCT**.*

We do not propose to defend this claim that **NRT** and **RCT** (and possible consequences therefrom) are cross-disciplinary commonalities, but will put the hypothesis to test by employing it, then viewing its consequences. Its proper defense will lie in its subsequent confirmation or disconfirmation.

> ***Proposition:*** *The phenomenon of communication is also in common to, and essential for all disciplines.*

Working Hypothesis 2: *Communication essentially incorporates a triadic relation.*

It appears plausible that communication is a process which relates a transmitting item through a message to a receiving item. Other ways of expressing similar structures are widely found. We speak of an Utterer, a Speech, and a Hearer – or of an Event, a Sign, and its Understanding. Peirce, in his **Semeiotic**, proposed a general pattern meant to present the relational form common to all such ways of speaking about the basic components of communication; the relation type that Communication exemplifies in form he called the ***Sign Relation***, or just the ***Sign***. The generalized components of a sign are:
its **Object** (what a particular communication process is about), its ***Representamen*** (a message-like feature representing some aspect of the object), and its ***Interpretant*** (a habit of some kind allowing understanding or comprehension of the object in terms of the representamen). This can be expressed in a single sentence as, in communication generally, An **O**bject is in a sign relation with a **R**epresentamen to an **I**nterpretant.

If this is expressed in Peirce's Rhematic notation, we have

___ is in a sign relation with ___ to an ___.

So we see that Peirce's Theory of Signs (Semeiotic) is a theory principally about a particular token of the triadic relation type he identified as a Sign Relation.

Now from **NRT** and ***Working Hypothesis 2*** one may conclude that a sign relation (the communication triadic relation which is a T type) cannot be constructed from only D types and the operation **B**.

It is now possible to incorporate a famous statement by Peirce into this sequence of development: "There is no exception, therefore, to the law that every thought-sign is translated or interpreted in a subsequent one [....]" *Collected Papers* 5.284 (1868).

We suggest that this means that if one wishes to understand more about some given sign relation, one will require additional communication about it – which will require more sign relations which are of the type **T**. That is, this suggests:

$\mathsf{T}_1]\![\mathsf{T}_2$ occurs, then understanding follows through an appropriate bonding operation (among the various kinds possible) between the two, the principal focus of understanding coming perhaps from the Interpretant of T_2.

So, perhaps another hypothesis is plausible, similar to Peirce's remark above.

Working Hypothesis 3: *Understanding of a given sign relation will require at least some other subsequent sign relation and some bonding connection between the given sign and the subsequent sign.*

One could also express a negative version: Understanding of a given sign relation cannot be exclusively in terms of D types only (using bonding), otherwise **NRT** would be violated. Attempts using *only* dyadic resources to study communication have often been tried, but if these working hypotheses are sustained, such an approach would be an inappropriate reduction.

These working hypotheses suggest a possibly fruitful path for cross-disciplinary study along the lines laid out in the science of Semeiotic.[8]

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Notes

1 For instance, "The Reader is Introduced to Relatives," *The Open Court*, volume 6 (1892): 3416–8. A number of additional references are to be found in *The Collected Papers of Charles Sanders Peirce*, Harvard University Press: Cambridge, 1931–1957. We will employ a terminology we have developed which we regard as based upon that used by Peirce. We regard this as an advisable step because Peirce constantly experimented with alternate terminologies throughout his career, the result sometimes being that similar ideas can seem to be different because new experimental terms appear.
2 Bonding, in our sense, will be defined below.
3 For example: Skidmore (1971), Freeman (1934), Hartshorne (quoted in Ketner 1989: 146–8), Quine (1966).
4 This is not an exhaustive list.
5 C.S. Peirce, "The Reader is Introduced to Relatives"; also published in *CP* 3.414f.
6 In keeping with Peirce's graphical approach to logic, we adopt 'Graph' instead of some alternate term, such as 'expression'.
7 'Weld' is a term Peirce often used to indicate that one item becomes continuous with a second item. See, for instance, Peirce 1992 (1898), 49, 91–92, 95, 158, 159, 160.
8 We distinguish sharply between Peirce's *Semeiotic* and contemporary *Semiotics* which often employs a reduction to only dyadic relations; that is, for that reason and many others, Peirce was definitely not a precursor of contemporary semiotics as is often claimed: see Fisch 1986, 321–355; also Percy 2002 (1959); Rochberg-Halton and McMurtrey 1983; Ketner 1981.