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PEIRCE AND PLATO

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I know little about Neoplatonism: I grasp that it is Platonic, and that it is Neo. It is a subject of interest to me, but I have spent the past few years some distance from Plotinus; I have been in the nineteenth century, trying to figure out a way to present the life and times of Charley Peirce. I hit upon a method, which is shown in my recent book.[1] Large parts of the method, if I may now take you into my confidence, are quite blatantly ripped off from the method in Plato's dialogues— chiefly his use of fiction to create a 'fallibilist' atmosphere. That, and the usual graduate school exposure to Plato, shows that I do have some acquaintance with Platonism.

Aha! Some sort of jackleg[2] cosmic dialectic is emerging here. Neoplatonism *is* Platonic, *and* it is little known that there *are* strong connections between Plato's work and Peirce's work. I can claim knowledge of Peirce's productions, and to a lesser extent, those of Plato, so I can put Plato and Peirce together in a way that might give you some resources for further study of their relationship, should you be inclined. *And,* I can leave it to your competent hands to add the Neo. That strikes me as being a sufficient transcendental deduction of how I came to be here among this distinguished group.

Most persons who have made any study of Peirce are aware that he described himself as being deeply immersed in the work of Kant during college and for a few years afterward—roughly 1855–1865. He

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1. K. L. Ketner, *His Glassy Essence: An Autobiography of Charles Sanders Peirce* (Nashville: Vanderbilt University Press), 1998.

2. Jackleg. This is a bootlegger's word. Legend has it that it began as a reference to a survivor of the War for Southern Independence, who had lost a leg during his adventures, but who had become a Tennessee bootlegger. His wooden leg was hollow, and he earned his living by smuggling good bourbon sippin' whiskey to his customers by means of his false limb. In southern dialect, it means *artificial.*

remarked that he almost knew the *Critic der reinen Vernunft*[3] by heart. He left behind this discipleship, however, when he discovered that his master had logical feet of clay. Perhaps "Feet of yogurt" would be a better literary figure to represent Peirce's disgust at the weakness in Kant's logic, a weakness that he discovered about 1865. From that discovery, Peirce launched into his own reinvention of logic, beginning with a series of papers in 1866–67 written for the American Academy of Arts and Sciences, among which was the famous path-breaking essay, "On a New List of Categories."[4]

However, there is more involved in this picture. Thanks to new material in *His Glassy Essence*, we can now see that Plato had a role in Peirce's intellectual blossoming. Here is an outline of the story. Peirce spent several months in the Gulf of Mexico on a U.S. Coast Survey schooner assisting in the survey of the east side of the Mississippi Delta and the area around Isle Au Breton. He returned to Cambridge by the end of May 1860, and by the end of June, he wrote an account[5] of what he took to be a continuous argument pattern running through the following Platonic dialogues: *The Apology of Socrates, Crito, Phaedo, Gorgias, Protagoras, Phaedrus, Theaetetus,* and *The Republic*. The argument that he hypothesized to be running across these dialogues was done up by Peirce in good Spinozistic style, including axioms, definitions, propositions, and scholia. Someone who is a real Plato expert should examine this project carefully, for it might be an original effort. One of these axioms, number 25, supposedly from the *Theaetetus*, sounds strangely like the ***Pragmatic Maxim***, which in Peirce's work first saw the full light of day, if I am right, in 1865.[6] Axiom 25 from Peirce's 1860 notes on Plato reads: "We only know our faculties by the conceptions or feelings they give us—by their results."

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3. Peirce did not follow contemporary translators who render the first word of Kant's book as *critique*. Peirce urged that Kant had written the word as the English term *critic*, after John Locke.
4. Ketner, *op. cit.*, 298–306, 328–338
5. *Ibid.*, 206–7.
6. *Ibid.*, 299ff.

This means that it is possible that Plato was an inspiration for Peirce's pragmatic maxim.

While we are on the subject of the *Theaetetus*, let me explain another possibly quite serious connection. Next to the pragmatic maxim, ***Semeiotic*** (his theory of signs) was Peirce's most important production. This is "See–my–OH-tick," not "Semi–AH–ticks." Semiotics is a contemporary endeavor bearing little relation to Peirce's well-worked-out Semeiotic, except perhaps for the fact that practitioners of semiotics often drop Peirce's name. Actually, one could argue that semeiotic was *more* important than the pragmatic maxim, for according to Peirce, the maxim is but a corollary of semeiotic.

What is semeiotic? It is usually subtitled "The Theory of Signs." But what is a sign? Here there is an ambiguity to be resolved, and those who don't resolve it are fated never to grasp Peirce's account of the matter.

Sign in the narrow sense is a representation. A Congress member represents home district voters to the central government. A stop sign represents a legally based command by a local government, that at this intersection, each motorist is required to stop, and then proceed with caution. To avoid confusion, Peirce often designated a representation, or a sign in the narrow sense, by the term ***Representamen***. Peirce was an advocate of using specially designed terminology in science so that results achieved by the current generation of scientific intelligences could be passed on without confusion to the next generation, thus facilitating a gradual increase of knowledge over the decades.

What is a ***Sign in the broad sense***? It is a triadic relation, a fact about three things. Peirce's logic of relations, which—as far as I can determine—is still the state of the art, depicts a monadic relation or property as a fact about one thing, a dyadic relation as a fact about two things, and so forth. He generalized this in the following table.

Jack is a Bolivian	**_____ is a Bolivian**
Sue hates Betsey	**_____ hates _____**
Tom promised $5 to Bob	**_____ promised _____ to _____**

Here, in order to display the structure, at the right hand of each sentence the subjects are generalized to a blank, which is understood to mean "place here whichever from the range of relevant things." On this analysis a sign in the broad sense, which we may also label as ***Sign Relation*** or ***Sign***, is a triadic relation, in this fashion: **This *Representamen* is a sign of this *Object* to this *Interpretant*. *Object*** is Peirce's term for that which the Triadic Sign Relation "is about." The ***Interpretant*** is whatever functions to interpret that the Representamen represents the Object. Peirce used ***Interpretant*** because he wished to leave open the question whether there are nonhumans who can perform such an interpretative function. Peirce thought that an exclusive use of *interpreter* would unduly restrict (prior to performance of relevant research) the range of possible interpreting functionaries only to human beings; he wanted to leave open the possibility that creatures other than humans (other species, Martians, intelligent devices, and so forth) could perform interpreting functions.

This brings us back to Plato. For Peirce, ordinary dialogue between human interpreters is a central guiding example of ***Semeiosis***, the action of Sign Relations. In human dialogue, there is a Speaker of a Message to an Interpreter. This occurs externally.

But, what of internal dialogue? Again, Peirce was inspired by the *Theaetetus*. Using Jowett's translation, as God intended me to do, I find the following passage.[7]

> *Soc.* And do you mean by conceiving, the same which I mean?
> *Theaet.* What is that?
> *Soc.* I mean the conversation which the soul holds with herself in considering of anything. I speak of what I scarcely understand; but the soul when thinking appears to me to be just talking—asking questions of herself and answering them, affirming and denying. And when she has arrived at a decision, either gradually or by a sudden impulse, and has at last agreed, and does not doubt, this is called her opinion. I say, then, that to form an opinion is to speak, and opinion is a word spoken—I mean, to oneself and in silence,

7. Plato, *The Dialogues of Plato,* vol. 4, trans. B. Jowett (London: Oxford University Press, 1871), 252.

not aloud or to another: What think you?
Theaet. I agree.

In 1868, Peirce remarked that this passage disclosed that external Semeiosis was dialogue and that the internal version of the same was too, and that in effect, thought or Semeiosis (either externally or internally) is dialogic in nature. In other words, in part through Plato's *Theaetetus*, he arrived at his famous conclusion that all thought is in signs. Here is how he presented the discovery in unpublished notes in 1868.[8]

It has been shown, then, by the preceding discussion that the only way of investigating a psychological question is by inference from external facts. I now proceed to apply this method of research to the solution of the question whether we can think otherwise than in signs.

Thought, says Plato [in the *Theaetetus,* as we have seen] is a silent speech of the soul with itself. If this were admitted, immense consequences follow; quite unrecognized, I believe, hitherto.... If we seek the light of external facts, we must certainly find only cases of thought in signs; plainly no other thought can be evidenced by external facts. It appears, then, that the only thought that can possibly be cognized is in signs....

From this proposition that every *thought* is a *sign* it follows that every thought must address itself to some other, must determine some other, since that is the essence of a sign. By the way, the word *determine* here does not mean *efficient cause;* it means *action to decrease indeterminacy in some indicated object of thought.*

The conclusion we can draw here was that Plato's work was a major catalyst for Peirce as he began to develop his own system of science within which semeiotic, the theory of signs, plays a dominant role. I believe that *His Glassy Essence* has provided the evidence for this possibility, which heretofore has not been properly emphasized within exegesis of Peirce's development.

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8. *MS* 931, in C. S. Peirce, *Writings of Charles S. Peirce: A Chronological Edition,* vol. 2: 1867–1871, ed. E. C. Moore et al. (Bloomington, IN, 1984), 172–173.

It is well known that Plato was the inventor of the first university prerequisite when he posted a sign over the entrance to the Academy that read, "Let none without geometry enter here." Peirce was as far away from being a-geometrical as it is possible to be. Perhaps this is another attraction Plato had for him. In any case, it is true that another major intersection with Plato lies in Peirce's conclusions about the nature of mathematics. This is well represented in a recently discovered and reconstructed set of lectures from 1898 that fulfills a long-standing need—a canonical book-length summary by Peirce himself of his late system. There is no space here to recount the fascinating story of how Peirce came to give these lectures, nor how they were lost, only to be found and reconstituted in 1992. One may follow these matters in the introduction to *Reasoning and the Logic of Things*.[9] There one will also find the lectures, along with Hilary Putnam's enlightening comments on each of them.

In fact, in this area, Peirce may have improved upon Plato. To see this possibility, one needs to grasp Peirce's account of reality. That there are real things, Peirce stated early in his career,[10] is a fundamental working hypothesis of the general scientific method. By the word Real, Peirce meant: That which has the properties it has independently of some one person's wish, hope, demand, command concerning those properties. It is important to be clear on this point. Another way to state Peirce's understanding of real is to say that the unreal is that the properties of which have been arbitrarily commanded, wished (and so forth) in an egocentric manner by some person. Or, one could express his understanding of real in an example: this one for instance. If in a chemistry lab, in a loud voice, I commanded (or hoped, or wished) some litmus paper to turn pink and on the basis of that action I concluded that it was pink, that would be a case of unreality. On the other hand, if I had hypothesized that the litmus paper in a crucial

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9. K. L. Ketner (ed.), *Reasoning and the Logic of Things: The 1898 Cambridge Conferences by Charles Sanders Peirce*, with an introduction by K. L. Ketner and H. Putnam (Cambridge: Harvard University Press), 1992. (= *RLT*)

10. In his well-known article, "The Fixation of Belief", in *The Popular Science Monthly* (1877), republished in *Collected Papers*, vol. 5.

experiment would turn blue, for if it did, I would win the Nobel Prize, but then noted that despite my wish or hope, it indeed turned pink when dipped, that would be a case of reality. Neither the real or the unreal is independent of person or persons, but real is independent of the mode of activity of ego functions such as hoping or wishing, whereas in the case of unreal, it is those arbitrary ego functions that precisely constitute the unreal. Or, consider another example. Suppose I have a dream. That the events in the dream happened because I dreamed them, is an example of unreality. However, it is real that I actually had that dream with those contents.

Peirce subdivided reality into two subclasses: real things that exist and real things that don't exist. Tables, chairs, rocks, and trees are examples of things that exist and which on Peirce's approach are indeed realities.

Systems, communities, relations, or necessary consequences are examples of nonexistent realities. For example, if I place my coffee cup on my study table, we can point to three 'reals': one is the existent cup, another is the existent table, but the third one is the nonexistent yet real relation of the cup to the table (in this configuration).

According to Peirce, mathematics is the science that discovers necessary connections between hypotheses. He regarded mathematical hypotheses as arbitrary in the sense that a mathematician is not concerned to determine whether the hypothetical material at hand actually represents anything in the everyday world. He proposed that persons engaged in mathematical inquiry only attempt to trace the consequences following necessarily from given hypotheses. This necessary "following from" is a relation, and if after a duly conducted inquiry we find—as a community of scientific intelligences—that ***Y*** really does follow from the hypothesis ***X***, then that "following from" is a real relation. Moreover, in mathematics such real relations or consequences of hypotheses are discovered, not created. Where then do such mathematical realities reside while they are not yet discovered? Three guesses, and the last two don't count: something like a Platonic

world of real potentials. Here is Peirce's description from his 1898 lectures.[11]

> Mathematics is based wholly upon hypotheses.... The host of men, who achieve the bulk of each year's new discoveries, are mostly confined to narrow ranges. For that reason you would expect the arbitrary hypotheses of the different mathematicians to shoot out in every direction into the boundless void of arbitrariness. But you do not find any such thing. On the contrary, what you find is that men working in fields as remote from one another as the African Fields are from the Klondike, reproduce the same forms of novel hypotheses.... All this crowd of creators of forms for which the real world affords no parallel...are, as we now begin to discern, gradually uncovering one great Cosmos of Forms, a world of potential being. The pure mathematician himself feels that this is so.... [I]f you enjoy the good fortune of talking with a number of mathematicians of a high order, you will find that the typical Pure Mathematician is a sort of Platonist. Only, he is Platonist who corrects the Heraclitean error that the Eternal is not continuous. The Eternal is for him a world, a cosmos, in which the universe of actual existence is nothing but an arbitrary locus. The end that Pure Mathematics is pursuing is to discover that real potential world.[12]

As a way to show how these Platonic influences have a considerable payoff, I will outline some recent thought of mine on Peirce's approach to mathematics. This should display other areas ripe for further research.

I have been trying to conduct a test of Peirce's categorical hypothesis, to determine if it will stand up to contemporary mathematics and logic. I began by noticing that the central bulwark and background of it is Peirce's logic of relations. In outline form, his master argument goes something like this.

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11. Ketner, *RLT*, 120–129.

12. I have earlier discussed these ideas on the ontology of mathematics in "Hartshorne and the Basis of Peirce's Categories", in *Hartshorne, Process Philosophy, and Theology*, ed. R. Kane and S. Phillips (Albany: State University of New York Press), 19. (= "HBPC")

Whatever we know, we know only by its relations, and in so far *as* we know its relations.[13]

In reality, every fact is a relation.[14] These considerations drive one to a study of the logic of relations. There, Peirce found reason for concluding that there are only three irreducible (or non-decomposable) forms of relation structure. Upon this basis, he erected a proof that monadic, dyadic, and triadic ***relational forms*** are all that one needs to construct every ***relational form***, and that triadic relations cannot be constructed from dyadic relations. This last proposition I named the Peirce-Percy Conjecture, because Walker Percy, the late, distinguished American philosophical novelist and essayist, came upon the same hypothesis independently of Peirce sometime during the 1950s.[15]

Next, I rummaged through any works on relational logic I could find. Not being an industrial-strength logician myself, I also sought out colleagues who were experts, then asked them whether Peirce's claims had been confirmed or disconfirmed in recent work. The answer I received from a number of them was, "Quine has shown Peirce was wrong." Eagerly I dashed off to the library to find Quine's "Reduction to a Dyadic Predicate",[16] wherein that result was said to be shown.

In this short article, Quine allows for the presence of a series of relations of any adicity whatsoever.[17] Then, with a tool kit he brought

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13. This is a direct quotation from Peirce, see *W*1:164.

14. Another quotation from Peirce at *P* 513, republished in *CP* 3.415.

15. See W. Percy, "The Delta Factor", *Message in the Bottle*, ed. W. Percy (New York: Farrar, Straus and Giroux, 1975), 3–45.

16. W. V. Quine, "Reduction to a dyadic predicate", in *Journal of Symbolic Logic* 19 (1954), 180–182; reprinted in W. V. Quine (1966), 224–226.

17. Adicity is a word of art within mathematical logic. If one focuses one's attention on the number preceding the "adic" suffix, one can speak of the 'adicity' of a relation. Thus a relation involving ten items has an adicity of ten (or is a decadic relation). In the numbers past four it seems to be more sensible in English to speak of adicity and then give some standard numeral: for example, a relation involving 15 items would be something like semidecadic or some strange word, so it is easier to say the relation has an adicity of 15. Peirce tended not to use adicity, which is a contemporary usage among mathematical logicians. He did, though, have the same concept, except he called it valency. So, in Peirce's language, our semidecadic relation would be a relation with valency of 15, or dyadic of valency 2, or quadradic of valency 4, and so

along, he purportedly shows how he can write the original series of relations as an equivalent series of dyadic relations. On the face of it, this exercise does seem to wreck Peirce's conclusion, for if one could reduce even one genuine triadic relation to dyads only, Peirce's claim would be false.

My first instinct was to examine Quine's tool kit, because if there is a triad hidden there, Quine could still be correct in whatever operation he was doing, but his work would not be a refutation of Peirce's non-reduction claim.

A principal feature among Quine's tools is the concept of ordered pair, the definition of which was given by Kuratowski in 1921.[18] Pairs sound quite dyadic, but I was able to show[19] that a triadic relation is concealed within the role the concept of an ordered pair plays within mathematical inquiry, and in the course of doing so, I also encountered one of the fundamental aspects of Peirce's ***Method of Diagrammatic Thought***. I will retrace my steps.

Begin at a common-sense level, and consider a set that is an unordered pair. Think of any two objects, abstract or concrete. It's obvious—isn't it?—that some person has to be imagining these two concrete or abstract objects in order for the two objects to become part of mathematical inquiry. Of course, if nobody performs this act of imagination, or if only one of us does, in the first place there will be no set for us to wrangle over, or, in the second place, no way for us to wrangle about *that* set, because we would have nothing in common for discussion. The most widely disparate items I can imagine are my Uncle Norvell's Zenith radio in the Arbuckle Museum in Murray County Oklahoma and the entrance gate of Saint Joseph's Abbey in Saint Tammany Parish Louisiana. This set enters mathematical inquiry when I bring these two items together in my imagination (and when you do the same in your imagination). We can discuss *this set* when

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forth. I suppose one could boil all this fat down and say that adicity is simply the count of the number of nodes of some particular relation type. But then, somebody is going to say, "What's a NODE?!" Such are the dangers of philosophical writing.

18. See K. Kuratowski and A. Mostowski, *Set Theory* (Warszawa, 1968), 59.

19. Ketner, "HBPC".

you and I are convinced that each of us is imagining the same thing. All this would be equally true, and even simpler to accomplish, if the two objects in the set were abstractions, perhaps variables such as ***a*** and ***b***, or numbers such as 6 and 329. This situation is captured in the sentence: *Ketner imagined a set composed of Norvell's radio and the Saint Joseph's Monastery gate.* Structurally this sentence is parallel to a sentence type one finds throughout books on set theory and math in general, sentences such as: *Let **a,b** be a set.* This simple little sentence, which is the first sentence type—a "Let..." sentence—in many proofs, in effect says: *You, reader, as I the writer have already done, imagine a set composed of variables **a** and **b**; do this so we can have a discussion about its consequences or features, so we can engage in mathematical inquiry.* Naturally, I am not saying that such sentences are only about sets; the point is that mathematical inquiry, which necessarily requires a dialogue or communication in order to function at all, typically begins with an invitation to imagine something, and imagination is an act of a person! Either of these sentences, however, is precisely a triadic relation incognito: _____ *imagined a set composed of* _____ *and* _____. This is indeed a fact about three things. Hence, any instance of *mathematical inquiry* that requires the notion of a set composed of two objects—a *pair*, or *unordered pair*—requires precisely a triadic relation in disguise, or better, in the background.

I have been accused of using sleight of hand to turn a dyadic relation into a triadic one, but the original dyad or pair is untouched by my point. It is not the dyad or pair that is triadic; but instead, it is the necessary and unavoidable act of imaging the dyad which is indeed a different relation—a triad. Mathematical inquiry is not possible without mathematical dialogue. Mathematical dialogue requires acts of imagination on the part of participants in such dialogue. Or to state this in the most direct manner possible—if, in a particular proof, a set of two, a pair, is required, then the following is also required in that proof: "The proof proposer (writer) imagined that ***a*** and ***b*** constitute a set pair." This last act of imagination by the proof writer is a triadic relation.

In his essay on reduction Quine used the concept of ordered pair, of course, and the new factor in this second concept lies in the order of the members: the set ***a;b*** would not be equivalent to the set ***b;a*** if both these sets were understood as ordered pairs. How does a *pair* become an *ordered pair?* Suppose we have two books on a table in front of us. I note their presence, imagine them as a set, then communicate that fact to everyone else present. Somewhat later I say something like this, *Let the book in the east be the first book listed in our set, and the book in the west be listed as second in our set.* If we describe the situation, we get: _____ *ordered* _____ *as first and* _____ *as second*, again a triadic relation. The result is that the introduction of an ordered pair into mathematical inquiry essentially requires a genuine triadic relation operating in the immediate background roughly constituted as an act of imagination by a person.

So, there are at least two triadic relations in Quine's tool kit which he did not openly enumerate; in other words, he has to say something like: "Dear Reader, imagine _____ and _____," as well as "Dear Reader, I order _____ as first and _____ as second." Thus, while Quine's inferences from his starting material, conducted by means of his kit of tools, are quite correct, this demonstration of his is not a refutation of Peirce's non-reduction claim. That is the case because Quine used these unlisted tool-kit triads to produce his result. This process violates the initial conditions of Peirce's claim; to refute Peirce, Quine would be allowed no triads, and only dyads, in his tool kit. Of course, Quine did not set out to refute Peirce; he set out to do what he finally did, whatever that might be. However, others have looked at his work and incorrectly judged it to be a refutation of Peirce's claim, an interpretation of Quine that one can show to be mistaken in the way just sketched. Now I began to think of the Peirce-Percy Conjecture as gaining support.

My humble efforts at supporting the Conjecture were about to be vindicated in a dramatic way. It was roughly at this point in my odyssey I met Robert Burch. He had noticed my earlier essay showing the relation in Peirce's thought between graph theory, valency, Existential Graphs, the categories, and Peirce's sense of

phenomenology.[20] We made contact and began to use our differing capacities to assist each other in our similar interests. Burch ultimately produced a revolutionary book—*A Peircean Reduction Thesis.*[21] Using more mathematically standard methods, he was able to vindicate Peirce's conclusions, and to show, in a way other than mine, using the highest level of mathematical rigor, that the Peirce-Percy Conjecture is correct and is *consistent* with Quine's results. Within my personal record-keeping system, I like to refer to this as the Peirce/Percy Principle, for it is no longer a conjecture: it has now been confirmed.

Meanwhile, in the course of my exercise about pairs, I stumbled across something of wider importance, something that I would later find to be an essential element in Peirce's account of diagrammatic thought.[22] I discovered one cannot accomplish anything in mathematics or logic unless there are at least two persons involved. Or, to put it bluntly, the ***P***s and ***Q***s, the sets, the variables, the numbers, the imaginaries, the limits, the lines and planes and solids, never dance alone at midnight on the autumnal equinox—they can't. As in Martin Buber's description of art, which requires a human to "body it over,"[23] mathematics and logic essentially and necessarily presuppose the intimate involvement of persons, and of communication between persons. (Communication, of course, is among the most genuine of triadic relations, and persons as intelligent beings are literally crawling with them.) Try to do some math or logic without some persons, or

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20. K. L. Ketner, "Peirce's 'Most Lucid and Interesting Paper': An Introduction to Cenopythagoreanism", *International Philosophical Quarterly* 26 (1986), 375–392.

21. R. W. Burch, *A Peircean Reduction Thesis: The Foundations of Topological Logic* (Lubbock: Texas Tech University Press, 1991).

22. It may be possible that Roger Penrose is onto something similar—consider the important role he assigns to consciousness, in physics of all things, in his recent book, *The Emperor's New Mind.* Bell's theorem, also a new finding in physics, appears definitively to show that there is no determinism at the quantum level (see Cushing's introductory essay in J. T. Cushing and E. McMullin (eds.), *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem* [Notre Dame, 1989], esp. 5–9), which if true would add further evidence to the outlook found in this here essay.

23. M. Buber, *I and Thou*, trans. W. Kaufmann (New York, 1970), 60–61.

communication between persons. It can't be done. Yet I'm convinced that a great many contemporary practitioners assume this feat can somehow be performed, and that the ***P***s and ***Q***s—and all the other notions, concepts, and entities—do dance alone in a fully self-sufficient and independent manner under a full moon at midnight, and that the factor of dialogue or communication between mathematicians can be ignored.

Am I describing a straw-person? If no one else can be found who exemplifies this factor, Quine certainly did, for he relegated all hints of involvement by persons to a category he called "phenomenology of mind," which he regarded as an irrelevancy in the formal sciences. I've heard him say it with my own ears. Very well, that is as good a name as any, but it misses the point. Mathematics or logic cannot happen without the "phenomenology of mind," without activities of persons. Some mathematicians make a similar claim of irrelevancy, but describe it by saying "the psychology of the practitioners of math is irrelevant to the objective results of mathematics." This is an obvious red herring, because my point is not a psychological one, but is instead a claim about the necessary presence — in order for mathematical inquiry to proceed—of acts by persons involving imagination and communication. Widespread belief in the self–sufficient metaphysical midnight lunar dance of the variables is probably one reason in my earlier search of the literature I rarely found any relation more complex than a dyad.

"Then you are a nominalist and anti-Platonist in regard to mathematical reality; for surely you hold that persons create mathematical entities by acts of imagination; you are also probably something like a Brouwerian intuitionist and likely hold that infinities are discounted because they cannot be imagined by finite human beings"—thus spoke an interesting objector to my previous comments. I think, however, my point was missed. This is ***not*** my claim: "It is not true that 'The entities dance at midnight'." Perhaps they do dance at midnight. This ***is*** my claim: "It is not true that 'The entities dance **ALONE** at midnight'."

Argumentation and evidence leads me toward something like mathematical Platonism. There is some kind of reality about which the dialogue of objective mathematical inquiry can converge. That is obvious on the basis of the history of mathematics.

But, let's leave open the exact answer to the vexed question of the nature of independent reality of mathematical entities. Whatever the source and nature of their reality, if we wish to study mathematics, we must first enter into a communication with a fellow scientific intelligence about these entities. Such a dialogue is required no matter whether the entities were recently created by a person or are somehow part of the eternal furniture of the cosmos. Furthermore, whatever mathematical notion, idea, or entity we wish to discuss must be conceived by the discussants; that is unavoidable—without mathematical imagination and dialogue there can be no mathematical inquiry. And, when the notion of a set that is a pair or an ordered pair comes into this dialogue, a triadic relation is present, as noticed above. We discard such a triad at the cost of discarding mathematical dialogue, and hence also the practice of math. We could say that dialogue is a part of the tool kit for every step in the formal sciences.

Perhaps the situation could be clarified by considering an analogy with chemistry. Imagine some simple experiment, such as mixing vinegar and baking soda together to make a fizz. The components of the experiment—vinegar, soda, a mixing bowl, pouring—are realities comparable to the furniture of a mathematical proof—variables, operators, valid hypothetical relations. In this experiment, a person has to imagine what to do and then do it. These acts of a person are in addition to the realities of the experiment components. The components do not self-perform the experiment. Likewise, no matter what the nature of the realities studied by mathematics, proofs (mathematical experiments, so to speak) do not self-prove. A proof writer who accomplishes various actions is required. Among these actions, we find triadic relations.

An objector remarked, "No one doubts the relevance of mathematical dialogue." The proper reply is, "Good, then we agree that

a triadic relation is essential to Quine's proof discussed above, one incorporating a set that is a pair or ordered pair."

And, I am no Brouwerian. We readily conceive infinity when we conceive ordinary arithmetic and the possibility of adding one more number to any number. Higher *orders of infinity*, *imaginary numbers*, *phase changes of probability-amplitude-matrices*, and all kinds of unusual mathematical entities are conceived when we comprehend the rules and procedures for forming these conceptions. Such examples only serve to increase the importance in mathematics of the essential role of imagination, and hence of persons.

My little discovery about the necessity of persons and mathematical communication will have no effect upon the actual practice of math or logic by those who describe themselves as doing otherwise, because even if some mathematician or logician claimed that persons and dialogue are not needed, such a claim would be immediately nullified in practice, as soon as somebody got around to doing some math. Thus, good math or logic would still be practiced, because theorems would be hypothesized by someone, and then proved by someone, and someone would elaborate formal systems. But, all that activity would happen in the presence of a crummy philosophy of math. Ours is not the first age (and math not the first subject) in which a plausibly functional practice has been informed by a defunct sub-rosa theory, as the history of other disciplines shows. This factor, however, has had every effect upon philosophy of math or logic; and denying its presence provides new bewitchments of our thinking on important topics. It particularly adversely affects general opinions widely held by formal and technical practitioners—persons who don't even set out to be philosophical. Rummage around in the formal sciences for yourself. I think you will find this little syndrome everywhere, and everywhere with unhappy philosophical consequences.[24]

There is no need to worry about philosophical consequences —right? Wrong! Unfortunately, these unconscious ones are the most

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24. A good basis for reflection in this general area is S. G. Shanker, *Wittgenstein and the Turning-Point in the Philosophy of Mathematics* (Albany, NY, 1987), especially ch. 6.

deadly kind. As I tell my students, bad thinking can kill. A brief reflection upon ancient or recent history is sufficient to confirm this sad fact. Perhaps the worst consequence of this little supposition lies in that section of the philosophy of the formal sciences in which people declare what such sciences can do for humanity. The school of Strict AI (Artificial Intelligence), with its constant but empty claim that in a few more years it will succeed in dyadicizing human intelligence, is an extreme and very illustrative limiting case. These folks are also devout about freestanding lunar sock-hops of the variables. Strict AI is an example showing that from such a little philosophical supposition an important body of potentially harmful consequential points can arise, not to mention large yet inherently doomed research projects funded at public expense.

As I come to the end of my allotted time, I see several items still on my list. In a longer presentation, I would want to describe how Peirce was an accomplished student of ancient philosophy, especially on its logical side. Among these accomplishments was an original effort on finding the order of the Platonic dialogues. He also spoke often and vigorously against the German critical school of his day, claiming that their logic was faulted, the logic whereby they made startling claims about ancient philosophic texts. He also developed an original and quite path-breaking (even in our contemporary time) methodology for historical work in general.[25] Then there is his general tone of fallibilism, which I think one can clearly show as being strongly inspired by the figure of Socrates in Plato's works. In a nutshell, Peircean fallibilism is the conclusion that any result of science is potentially open to revision. It is not a property of sentences or of theories, but is a property of persons, roughly what we call "having an open mind."

But all of these will have to wait for another day, because, as Cousin Minnie Pearl, on Nashville's GRAND OLE OPRY, often said in recent years at the end of another memorable Saturday night radio

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25. C. Eisele (ed.), *Historical Perspectives on Peirce's Logic of Science: A History of Science* (Berlin, 1985).

broadcast of authentic American country music and humor: "I'm going to the wagon, 'cause my shoes are killin' me." If I may be permitted to translate that remark from dialectical American into contemporary standard British, it means: "I will be stopping now, to return to my dwelling, because after all this work, I require the services of a podiatrist or reflexologist to ease the work-induced pain in my lower extremities."